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Resonance Constraints on Rhythmic Movement

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The component frequencies of rhythmic patterns forming rational ratios, either simple (e.g., 1:2, 1:3) or complex (e.g., 2:3, 2:5), are known as mode locks or resonances. A general theory of resonances is provided by the circle map, the Farey series, and continued fractions. Predictions were evaluated in which rhythms (simple and poly) were established implicitly—the subject neither intended them nor knew their ratios. In Experiment 1, a prescribed unimanual frequency was performed as the primary task while hearing another frequency irrelevant to the task. In Experiments 2 and 3, a hand-held pendulum was oscillated at its natural frequency, while the other hand performed the primary task of following a metronome. The frequency ratio at the outset of a trial often changed during the trial. Consistent with the general theory, shifts were toward unimodular ratios of the Farey tree, and Fibonacci ratios tended to shift more than non-Fibonacci ratios.

Rhythmic performances involve patterns of varying complexity. Polyrhythms (ratios such as 2:3, 2:5, and 3:5) are periodic patterns in which the respective events of the component rhythmic units coincide only once per cycle. For example, a 2:3 polyrhythm involves two isochronous pulse trains such that the first completes two events in the same duration that the second completes three events. Despite their prominence in the music of a number of non-Western cultures (Chernoff, 1979; Locke, 1982), incompatible rhythms are difficult to produce, especially in comparison to simple rhythms (ratios such as 1:1, 1:2, and 1:3; Ibbotson & Morton, 1981; Klapp, 1979, 1981; Klapp et al., 1985; Peters, 1977, 1980, 1981, 1985a, 1985b, 1987; Peters & Schwartz, 1989), and fluctuations in polyrhythmic coordinations amplify with ratio complexity (Deutsch, 1983). It has been suggested that these observations on human rhythmic behavior are explained by the hypothesis that complex rhythmic movement patterns are distinguished by the number of chunks required to code them (Deutsch, 1983). For example, a bimanual polyrhythm of 2:5 would be internally represented as [R/L—R—R L R—R—], where R = right, L = left, R/L = right and left simultaneous, and the dash specifies a pause. In contrast, a polyrhythm of 2:3 might be coded less complexly as [R/L—R L R—]. Some investigators have tested whether polyrhythmic performance involves the use of a type of motor organization based either on two independent parallel codes or on a single integrated code (Jagacinski, Marshburn, Klapp, & Jones, 1988; Klapp et al., 1985; Summers & Kennedy, 1992). Because variability of performance increases with both ratio complexity (Deutsch, 1983) and intertap covariation (Jagacinski et al., 1988), some form of parallel integration is implied in these tasks.

In this article we explore the possibility that a physical basis underlies such an integrated motor organization based on general principles of mode locking in nonlinear dynamical systems. In contrast to other studies, we examine performance under circumstances whereby the two limbs producing the polyrhythmlike movement need not both be intentionally controlled by the subject. Although distinct from intentionally produced polyrhythms, such unintended polyrhythmlike behavior may provide insight into a generic means of control shared by both the intended and unintended classes of action. In the next two sections we identify these general principles and then proceed, in the remainder of the introduction, to identify the major predictions about simple and polyrhythm behavior that follow from them and a basic methodology by which the predictions can be evaluated.

Circle Maps as Characterizations of Frequency Entrainment

When observing pendulum clocks mounted on the same wall, one notes that when the frequency of one oscillator is close to that of another, entrainment can occur whereby one oscillator becomes synchronized to the frequency and phase of the other. Such mode locking (frequency locking, phase locking, or both) is characteristic of nonlinearly coupled oscillators of two or more competing frequencies. A mode lock constitutes a resonance or attractor (Abraham & Shaw, 1992; Jackson, 1989); one frequency may be modified in the presence of another, competing frequency to yield a new, shifted frequency ratio.

The damped, driven pendulum is a classic example of a nonlinear dynamical system that may be examined by numerical integration of the underlying continuous differential equation. We use the more transparent discretized version of the system in the form of an iterated difference equation. This

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permits the state of the driven oscillator to be seen at discrete points in time at a particular periodic phase of the driving oscillator. The phase space for such a system of two coupled oscillators is the cross-product of their two limit cycles, topologically equivalent to a 2-torus (a doughnut-shaped manifold), as depicted in Figure 1. Pictorially, the driver frequency winds around the major axis of rotation of the torus while the driven oscillator correspondingly winds around the second, perpendicular minor axis of the torus. If the two oscillators are $p:q$ phase locked (e.g., 1:3 or 2:3), then as the orbit of the driver oscillator winds p times around the torus, its trajectory will always be intersected at the same point every q successive cycles of the driven oscillator. If the trajectory closes on itself after an integer number of cycles, then periodic motion occurs. If the trajectory fails to close on itself, but comes arbitrarily close to its previous orbits, then there is phase drift, and the motion is called quasiperiodic, with the trajectory eventually covering the complete surface of the torus.

If the helical trajectory of the forced oscillator on the torus surface is observed or "strobed" at intervals of the driver frequency, the resultant diagram, known as a Poincaré section, yields information on the resonance dynamics. For phase-locked systems, the Poincaré section exhibits discrete points distributed around the circle, the number of which corresponds to the periodicity of the driven oscillator. That is, the Poincaré section produces an iterative one-dimensional discrete-time map of the closed curve in the form of a circle. It is these iterated-map models of mode locking that are known as *circle maps*. They are defined using the phase angle of the forced oscillator, θ_t, measured at the strobed intervals, $t_n = 2\pi n/\omega$:

$$\theta_{t+1} = f(\theta_t) = \theta_t + \Omega + g(\theta_t), \qquad (1)$$

where

$$g(\theta_t) = g(\theta_t + 1) \text{ (modulus 1)}.$$

From the above, the new phase θ_{t+1} is a function f of the old phase, the ratio of the uncoupled frequencies ($\Omega = p/q$), and

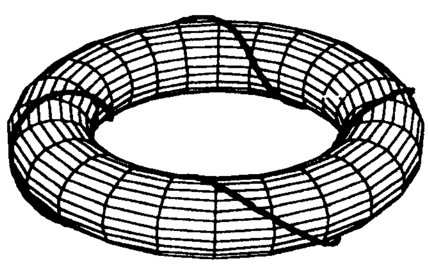

Figure 1. Geometrical representation of the dynamical behavior of two coupled oscillators, mode locked in a 1:4 (p/q) resonance. (The mode-locked dynamics may be said to occur on a manifold generated by the Cartesian product of two circles, that is, a 2-torus. As the slower, driver oscillator (p) winds around the major axis of the torus, the faster, driven oscillator (q) simultaneously winds around the minor axis of the torus. A 1:4 resonance is here defined because after four cycles, the faster oscillator intersects its own trajectory at the precise point at which, after one cycle, the slower oscillator intersects its trajectory.)

a nonlinear coupling between oscillators, $g(\theta_t)$. The coupling $g(\theta_t)$ is periodic such that θ_{t+1} (modulus 1) represents a complete phase rotation (Jensen, Bak, & Bohr, 1984).

As the circle map is iterated, the dynamics of the driven oscillator are summarized by the "winding number," W, equivalent to the average number of rotations or average phase advance per iteration. The dynamics of the driven oscillator may be either periodic, with a rational winding number equal to p/q defined as $W(K, \Omega) = (\theta_t - \theta_0)/t$ in the limit as t approaches infinity; or quasiperiodic, with an irrational winding number. Computational implementations have usually been of the "sine circle map" and have taken the form

$$\theta_{t+1} = f(\theta_t) = \theta_t + \Omega + (K/2\pi) \sin 2\pi\theta_t, \qquad (2)$$

where the coupling, K, acts as a control parameter for the extent of mode locking. As K increases under constant Ω, corresponding, for example, to a larger amplitude of forcing, the size of a given resonance region with $W = p/q$ increases even though the frequency ratio of the individual uncoupled oscillators Ω may not itself be rational. Thus, when in a resonance, a rational W characterizes frequency-locked dynamics of, for example, exactly 0.5 (1:2) even though the two individual frequencies may have an irrational ratio p/q of 0.51327. For an appropriate selection of K and Ω control parameters, there exists a finite interval or "window" of mode locking with $W = p/q$. Such stability intervals act as attractors of nearby trajectories and when plotted in a regime diagram (a plot of the preferred modes) with coordinates of a control parameter (K) and frequency detuning (Ω), they are known as *Arnold tongues* (Glass & Mackey, 1988; Jackson, 1989; Schroeder, 1991); they are shown in Figure 2. Between every mode-locked Arnold tongue lies a quasiperiodic region. The widths of these quasiperiodic regions decrease as K increases; when $K = 1$, quasiperiodic regions are "squeezed out" and mode locking exists for every choice of Ω. Beyond the $K = 1$ line, the resonances necessarily overlap, which may result in deterministic chaos. This means that the behavior of the system involves hysteretic shifting between resonances for different but arbitrarily small initial phases (Jensen et al., 1984; Schuster, 1988). Although apparently random, such dynamics are completely determined by the equation of motion.

Farey Tree Description of Resonance Layout

A central feature of both numerical and empirical mode-locking experiments is the ability to characterize the pattern of mode locks by an elegant number theoretical construction called the *Farey series* (Hardy & Wright, 1938; Haucke & Ecke, 1987; Maselko & Swinney, 1985; Schroeder, 1984, 1991; Stavans, 1987). Given two ratios p/q and p'/q' as "parents," the Farey mediant lying between the two parents is defined as the ratio $(p + p')/(q + q')$. A hierarchical structure called the *Farey tree* can thus be generated beginning with parents 0/1 and 1/1. Figure 3 shows all ratios from the first five levels of the tree. The pattern of resonances observed experimentally has been shown to correspond precisely to the Farey series in that wider resonance regions are

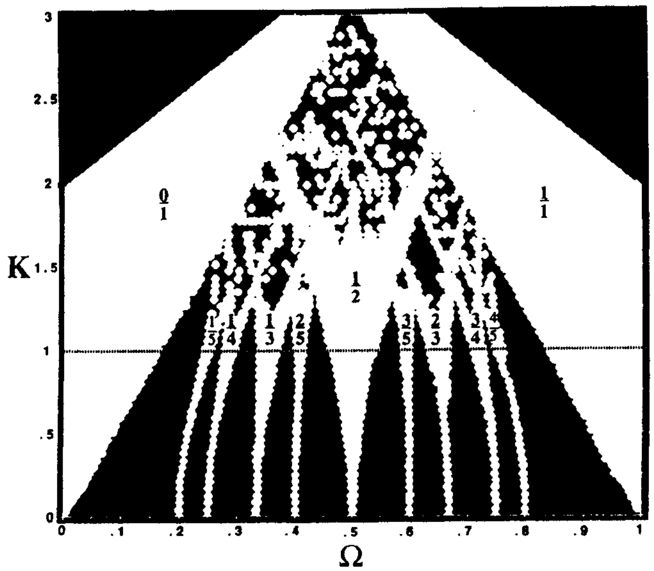

Figure 2. Regime diagram generated by the sine circle map for uncoupled frequency ratio, $\Omega = 0$ to 1, and coupling strength, $K = 0$ to 3. (Resonances or Arnold tongues [open areas] are labeled with mode lock, or winding number, $W = p/q$ [only Ws with $q \leq 5$ are shown]. Quasiperiodic regions are indicated by solid areas. Note that above $K = 1$ [indicated by the horizontal line], the resonances begin to overlap, and mode locking becomes unavoidable. The apparently random scatter above $K = 1$ indicates that stable mode locks still exist apart from the large, orderly regions.)

composed of ratios drawn from lower order levels in the tree. Thus, the Level 1 ratio of 1/2 is a wider resonance than the Level 2 ratio of 2/3 (see Figure 3).

There also exists a strong relation between any two adjacent ratios p/q and m/n, such that $|p*n - q*m| = 1$. This unimodularity or "mod 1" condition (Allen, 1983; Coxeter, 1961; Hardy & Wright, 1938; Weyl, 1980) defines the property of ratio adjacency (see the Appendix for the geometrical significance of unimodularity). For example, 2/3 and 1/2 are unimodular ratios ($|2*2 - 3*1| = 1$), whereas 2/3 and 4/5 are non-unimodular ($|2*5 - 4*3| = 2$). Because of their isomorphism to the Farey tree ratios, adjacent Arnold tongues also conform to the unimodularity condition (Figure 2). Experiments with physical dynamical systems have shown that bifurcations between resonances follow a route satisfying mod 1 relations in the Farey tree rather than some more arbitrary sequence of shifts (Maselko & Swinney, 1985). Hence, unimodular shifts may provide a means of accessing states of dynamical optimality (Levitov, 1991).

By providing a systematic enumeration of all integer ratios with the denominator not exceeding a given value, the Farey series and tree have great generality in terms of the periodic phenomena they may capture, including perception of consonance in detuned musical intervals (Hall & Hess, 1984) and, correspondingly, the diatonic musical scale (Révész, 1954). The circle map, as the deterministic generator of the Farey series, has predominantly been used as a means of describing one-way forcing or coupling in both physical and biological systems such as hydrodynamic flows (Fein, Heutmaker, & Gollub, 1985; Stavans, 1987), chemical and acoustic oscillations (Maselko & Swinney, 1985; Yazaki, Sugioka, Mizutani, & Mamada, 1990), heartbeat (Courtemanche, Glass, Belair, Scagliotti, & Gordon, 1989; Guevara & Glass, 1982), neural dynamics (Allen, 1983; Bressloff & Stark, 1990), human cascade juggling (Beek, 1989), 1:1 synchronization of limbs (Schmidt, Beek, Treffner, & Turvey, 1991; Beek & Turvey, 1992), and also polyrhythmic finger tapping (Beek, Peper, & van Wieringen, 1992; deGuzman & Kelso, 1991; Kelso & deGuzman, 1988; Peper, Beek, & van Wieringen, 1991). Circle map dynamics have also been shown to provide an accurate characterization of systems with reciprocal forcing, such as two intrinsic oscillatory modes that coexist in a hydrodynamic flow (Haucke & Ecke, 1987) or rhythmic finger coordination (Kelso, deGuzman, & Holroyd, 1991). For this reason we consider the circle map an appropriate model for bimanual tasks that may not involve unidirectional coupling.

Farey Tree Predictions for Coordinated Rhythmic Movements

The resonance regions (Arnold tongues) in the regime diagram depicted in Figure 2 exhibit markedly different widths for different winding numbers, W, at some constant coupling strength, K. It may be inferred that the width of a resonance determines the probability that a given coupled oscillator system remains at a particular W under random perturbation such as rhythmical fluctuations or "noise." This has been explored using noisy circle maps (Glass, Graves, Petrillo, & Mackey, 1980; Markosová & Markos, 1989) and may characterize the stability observed in biological, self-organized systems (von Holst 1939/1973; Scholz, Kelso, & Schöner, 1987). It may be concluded that given a random perturbation, wider resonances will afford more stable behavior than narrower resonances. In the regime diagram of Figure 2, the

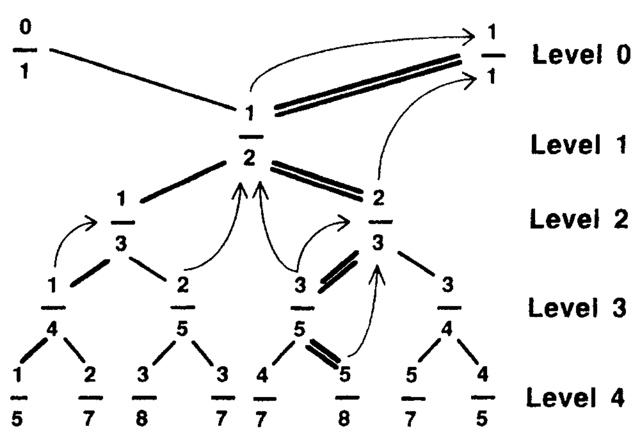

Figure 3. The first five levels of the Farey tree depicting ratios, p/q, from lower order mode locks (Level 0) to higher order mode locks (Level 4). (Also indicated is the path of simple ratios down the left-hand side, and the path of Fibonacci ratios [double lines]. Several ratios from which unimodular shifts may occur are also indicated.)

largest and consequently the most stable Arnold tongue is also the biologically predominant 1:1 absolute coordination mode of terrestrial locomotion, which is accessible across the complete range of Ω ratios by an appropriate selection of coupling strength (Schmidt et al., 1991). In contrast, as one moves down the Farey tree toward higher order levels, increasingly less stable and, importantly, less biologically prevalent mode locks are encountered.

The resonances act as attractors for nearby trajectories of the dynamical system. Because of differences in width, however, resonances with ratios drawn from lower order levels (e.g., Level 1 or 2) will act as stronger attractors than the narrower resonances from higher order levels. Thus, it is to be expected that in the presence of small phasing fluctuations common to rhythmic movement tasks (Rosenblum & Turvey, 1988; Turvey, Schmidt, & Beek, 1993), the system will be perturbed away from less stable ratios and toward neighboring ratios (with unimodular relation) more often than to some other ratio (with non-unimodular relation). If true, this would support the hypothesis that a particular geometry of resonances as partitioned by the Farey tree underwrites the stability and shift phenomena of rhythmic movements.

We thus make a general prediction for polyrhythmic movements that conform to these dynamics: Because the final ratio produced (which we call "*W*-actual") tends to be different from the initial ratio ("*W*-expected"), the limbs will not, in general, perform some arbitrary pattern but rather will be attracted toward a neighboring, unimodular ratio as implied by the geometry of mode locking (Figures 2 and 3; see also the Appendix). We call this the *unimodularity shift hypothesis.*

Dynamical Systems Studies of Simple and Polyrhythmic Movement Patterns

In a preliminary study applying the theory of nonlinearly coupled oscillators to polyrhythmic movement, Kelso and deGuzman (1988) required subjects to track a visual metronome with the finger of one hand. The signal was then switched off, and the corresponding finger of the other hand was simultaneously driven by a mechanical oscillator at a frequency different from that of the finger that continued the initial rhythm. Six ratios were generated, two simple and four polyrhythmic: 1:2, 1:3, 2:3, 2:5, 3:4, and 3:5. The standard deviation of the frequency of the nonforced finger was measured and found to increase under different ratio conditions. It was found that when the ratio was either 2:3 or 3:4, the actual frequency produced moved in the direction of 1:1, and a frequency ratio of 2:5 shifted toward 1:3. The simple ratios were found to be less variable than the polyrhythms, as in Deutsch's (1983) study. The 1:1 ratio was interpreted as a strong dynamical attractor for nearby ratios such that bimanual behavior asymptotically approached a 1:1 regime. In addition, a learning effect was seen during a second block of trials whereby 3:4 was performed more consistently (i.e., less influenced by the 1:1 ratio). Sensitivity to the attractor layout was thus influenced by practice and skill level. In a similar vein, Peper et al. (1991) found that highly trained musicians modified a 2:5 or 3:5 tapping polyrhythm when a pacing metronome was increased in frequency (cf. Kelso, Schöner, Scholz, & Haken, 1987; Schmidt, Carello, & Turvey, 1990). The change was shown as compatible with a bifurcation sequence predicted from the Farey series: 2:5 shifted to 1:2, and 3:5 shifted to either 1:2 or 2:3 (Figure 3). Significantly, these sequences conformed to the unimodular structure of the Farey tree.

Identifying a Model Task

To test the hypothesis that resonance constraints provide the foundation for coordinated rhythmic movements, one needs a task that makes performance as free as possible from the confounds of nondynamical factors. Moreover, a task is preferred within which simple rhythms and polyrhythms are performable by novice subjects without any specialized training in music or the rhythmic arts. There is one further requirement. Previous investigations of polyrhythm perception and production have used an audible or visible polyrhythmic pulse train to establish the desired performance and have suggested that polyrhythm production may have a significant perceptual component (Beauvillain, 1983; Beauvillain & Fraisse, 1984; Handel, 1984; Handel & Lawson, 1983; Handel & Oshinsky, 1981; Jagacinski et al., 1988; Klapp et al., 1985; Klapp, Porter-Graham, & Hoifjeld, 1991; Pitt & Monahan, 1987; Summers, Bell, & Burns, 1989). The pacing stimulus should be distinct from the desired polyrhythmic movement pattern to avoid a possible confound between polyrhythm perception and polyrhythm production, and it should also be readily detectable (Kolers & Brewster, 1985). In the present article we explore two paradigms that satisfy the preceding requirements of a model task. Paradigm A is a unimanual task in which a frequency of oscillation for a hand-held drumstick is established by an auditory metronome, the metronome is turned off, and the rhythm is continued by repetitively beating the drumstick in midair while simultaneously hearing a second, different frequency through earphones. In this situation the auditory signal may be considered as the "driver" oscillation, and the continued drumstick tempo may be considered as the "driven" oscillation. Paradigm B is a bimanual task in which a hand-held pendulum is oscillated continuously at the subject's preferred frequency, while a different frequency, determined by an auditory metronome, is tracked simultaneously with either a pendulum or a drumstick in the other hand. In Paradigm B, it is arbitrary as to which hand is considered the driver and which the driven oscillator because the assumptions of mutual forcing hold. The general dynamical theory of mode locking leads to the unimodularity shift hypothesis: When performing a polyrhythmlike movement, the subject will tend to change unknowingly the frequency of the drumstick in Paradigm A and the comfortably oscillating pendulum in Paradigm B to a frequency that satisfies a unimodular ratio relative to the initial one. This means that the frequency of drumstick or pendulum will change so as to result in a shift in the Farey tree, specifically toward a ratio—of drumstick and metronome frequency in Paradigm A, and of pendulum and other-hand frequency in Paradigm B—that bears a unimodular relation to the initial ratio.

The measure of simple rhythmic and polyrhythmic performance in Paradigms A and B differs from more conventional measures. The measure of satisfactory reproduction of a polyrhythm in the finger-tapping paradigm is the interleaving of the individual taps in the precise manner required to yield an exact polyrhythmic ratio during each bimanual cycle. Complying with this measure proves demanding, even for trained musicians (Peper et al., 1991). The experiments reported in this article involve a less restrictive measure but one motivated by circle map theory. In correspondence with the previous definition of the winding number (W) as the average phase advance per unit time, or average frequency, of the coupled system, a produced rhythmic pattern is evaluated as the average frequency ratio performed over the full duration of a trial. Importantly, this measure also conforms with the observation that two coupled biological rhythms exhibit phase *entrainment* rather than phase *locking*, meaning that the phase angle of one covers a full cycle in the same amount of time as the phase angle of the other without necessarily maintaining a fixed phase difference during the cycle (deGuzman & Kelso, 1991; Kelso et al., 1991; Schmidt, Shaw, & Turvey 1993; Sternad, Turvey, & Schmidt, 1992; Turvey et al., 1993). Indeed, the best term for biological rhythmic coordinations is phase *attraction* (deGuzman & Kelso, 1991; Kelso et al., 1991; Kelso, DelColle, & Schöner, 1990). In the original work of von Holst (1939/1973) on the medulla-transected *Labrus*, contrasting interfin patterns of absolute coordination (1:1 frequency locking and maintenance of a fixed phase relation) and relative coordination (unlocked frequencies but a tendency to certain phase relations) were observed in any given time series. Kelso and colleagues (deGuzman & Kelso, 1991; Kelso, 1991; Kelso et al., 1991) have furnished a nonlinear dynamical analysis that reveals a connection between the relative phase observed by von Holst and the phase intermittency associated with tangent bifurcations (Manneville, 1990).[1] With respect to von Holst's medulla-transected *Labrus*, a pair of fins of unequal natural frequencies in relative coordination would hover in the vicinity of 1:1 frequency and phase locking for some time, wander fleetingly through other phase relations, once again return and stay in the vicinity of 1:1, and then wander off again. This cycling of phase accords with the intermittency associated with a tangent bifurcation.[2]

Understanding that the dynamics of biological movement systems are phase attractive has significance for the present research and related studies. It suggests how the hierarchy of Farey ratios may provide one and the same attractive stability structure for systems that exhibit rigid phase locking and for those, such as biological movement systems, that exhibit flexible yet stable phase patterns by behaving close to phase-locked regions without necessarily settling into them (Beek, 1989; Kelso et al., 1991; Turvey et al., 1993).

Returning to the model task, the central idea behind it is that of "preparing" a subject at the outset of a trial in a given rhythmic pattern without the subject knowing the pattern in advance and without the subject having any explicit responsibility for the preservation of the pattern. The idea is to impose, as effortlessly as possible, competing frequencies on the perception–action system. To achieve this requires that a subject's attention is not directed explicitly at the performance of, for example, a 3:5 pattern. Rather, it is directed at the 3:5 pattern only implicitly, through the instructed goal of keeping one of the hands oscillating at a prescribed frequency. Maintaining the prescribed hand frequency is the focus of the subject's explicit attention. For this reason the movement pattern is better described as polyrhythmlike rather than strictly polyrhythmic. Guiding the application of Paradigms A and B is the assumption that, in whatever way competing frequencies are defined on the perception and action subsystems, the persistence of the ratio they form will be governed systematically by resonance constraints. The involved neural and muscular mechanisms must interact, and their interactions must comply with general dynamical principles. Accordingly, the proposed measure of frequency ratio averaged over a trial should provide, in its departures from the original "prepared" ratio of competing frequencies, a basis for evaluating the unimodular shift hypothesis.

Experiment 1

Listening to a rhythmic pattern and, concurrently, moving rhythmically are regular human achievements. It is apparent that there exists a coupling between the temporal pattern listened to and the temporal pattern of bodily motions produced. Equally pronounced is the appreciation that executing a single-handed tempo different from that listened to can be a challenging task (Klapp et al., 1985). However, trained musicians rarely maintain a fixed tempo (Shaffer, 1981). It has been argued that the dynamic event structure of the rhythmic context determines the selection of particular temporal structures over others (Jones, 1990; Jones & Boltz, 1989; Jones, Kidd, & Wetzel, 1981). In the first experiment, Paradigm A is applied to assess whether the unimodular geometry is selective of particular ear–hand coordinations. Essentially, the subject in the course of producing the hand frequency f_{hand} hears a metronome frequency $f_{metronome}$, where initially

[1] Weakly coupled oscillators whose natural frequencies compete will be attracted toward integer frequency locks even though they cannot perfectly attain them. Graphically, the time derivative of phase as a function of phase may exhibit no zero crossings and hence no stationary fixed points (stable or unstable). However, the curve may tangentially approach the phase axis such that there exist only trajectories that gradually approach and veer away from the very zero crossings that would normally be attained with greater coupling or oscillators of more closely matched natural frequencies (Kelso et al., 1990, 1991). Such a point still provides phase attraction by means of the "ghost" of a fixed point (Manneville, 1990) even though the fixed point proper has disappeared (the curve has lifted off the axis). Under these conditions, periods of coherent, laminar behavior with steady phase will occasionally be interrupted by bursts of incoherent phase, phase wandering, and (possibly) chaotic behavior.

[2] Although intermittency, with quasiperiodicity, are two of the bifurcation routes to chaos exhibited by dynamical systems, only if some control parameter (such as K in the circle map) is scaled continuously may a bifurcation to chaos be obtained.

$f_{hand} \neq f_{metronome}$, such that the produced and heard frequencies are in competition. The question posed is, Will an intended manual rhythm f_{hand} be deflected (spontaneously) by an auditory rhythm $f_{metronome}$ in the direction of forming a new, shifted ratio f_{hand}:$f_{metronome}$ that is in unimodular relation to the initial ratio? That is, will the final ratio, *W*-actual, be in a unimodular relation to the initially produced ratio, *W*-expected?

To perform this analysis it was necessary to decide whether a value of *W*-actual was sufficiently close to a rational number to be considered equivalent to the latter. Previous studies have determined a value for the performed ratio, *W*, through either subjective coding of *W*-actuals (Peper et al., 1991) or designation of a fixed parsing window, for example, ± 0.05 (Paterson, Wood, Marshall, Morton, & Henstridge, 1986). In the present method, *W*-actuals were computationally parsed using very conservative tolerance windows on either side of a given rational *W*. The ranges were generated using the sine circle map (see Equation 2) to specify the widths of all mode-locked Arnold tongues from the first six levels of the Farey tree (Levels 0–5). In addition, the regions on either side of a ratio were required not to overlap another ratio from the same level or any other of the top six levels of the tree. The value of *K*, the coupling strength in the circle map, was set to 1 to generate the tolerance windows. Because the relative widths of the Arnold tongues are maintained over the range $K \leq 1$, $K = 1$ offered appropriate widths of the tolerance windows prior to the onset of the overlap of resonances and chaos. On either side of a *W*-ratio, the windows were Level 0 (i.e., 1:1), 0.0714; Level 1 (i.e., 1:2), 0.0396; Level 2, 0.0172; Level 3, 0.0076; Level 4, 0.0036; and Level 5, 0.0015. Thus, a *W*-actual of 0.535 would be parsed as *W* = 1:2 because the Level 1 window for 1:2 is given by 0.5 ± 0.0396, which encompasses 0.535.

Because the tolerance windows decrease in width as Farey level is increased, a measure of performance is required that is uninfluenced by the differential widths. To this end, the analysis focused on the issue of whether, in general, more mod 1 than non-mod 1 shifts occurred for a given *W*-expected (the unimodularity shift hypothesis). That is, were the shifts patterned according to the structure of the Farey tree (mod 1), or were the shifts arbitrary with respect to final ratio settled on (non-mod 1)? Because the dependent measures of proportions of mod 1 and non-mod 1 shifts involve shifts to various ratios from various Farey levels, they are not biased by differential window widths. With the exception of Levels 0 and 1 (1:1 and 1:2), each level of the Farey tree contains either mod 1 or non-mod 1 ratios relative to some initial *W*-expected from another level. However, Levels 0 and 1 do not possess this property because their single, component ratio rules out a mod 1 or non-mod 1 counterpart. Consequently, in deriving the proportions of mod 1 and non-mod 1 shifts, any shifts toward 1:1 or 1:2 must be excluded from the analyses because Level 0 or Level 1 can provide only either a mod 1 or a non-mod 1 ratio for some initial *W*-expected.

Because the unimodularity hypothesis states that more mod 1 than non-mod 1 shifts will occur if Farey constraints apply, it is necessary to compare the experimental shift data to what would theoretically be expected on the basis of chance behavior. However, the Farey tree generates an infinite number of ratios, and so estimation of the expected probabilities of mod 1 and non-mod 1 shifts was limited to only those ratios from the first six Farey levels (Levels 0–5). Table 1 presents, for each *W*-expected of the present experiments, the number of mod 1 and non-mod 1 resonances and the percentage of the *K* = 1 line covered by them as determined by summing the sizes of their respective tolerance windows (consider again the Arnold tongues depicted in Figure 2). Table 1 reveals that there are many more non-mod 1 ratios than mod 1 ratios (consider, once again, Figure 3) and that non-mod 1 ratios cover much more of the *K* = 1 line than mod 1 ratios. Consequently, *W*-actual is more likely to be parsed as a non-mod 1 ratio than as a mod 1 ratio. For example, if a shift occurs from *W*-expected = 4:7, the chance of the *W*-actual being parsed in the analysis as a non-mod 1 ratio rather than as a mod 1 ratio is approximately 10:1 (20.8% vs. 2.1%). The hypothesis under investigation counters this greater chance expectancy of non-mod 1 shifts: It expects that shifts should be predominantly of the mod 1 kind because such shifts are constrained by the unimodular structure of the Farey tree.

Method

Subjects. A total of 25 right-handed people (9 men and 16 women) participated in the experiment. They were graduate and undergraduate students at the University of Connecticut.

Materials. A chair was used that had an attached right-hand writing surface on which the subject could rest his or her arm comfortably. An electronic auditory metronome that emitted short-duration blips was positioned 150 cm behind the subject's seat. A tape recorder and earphones were used to present previously recorded metronome frequencies. The intensity of the binaural presentation was comparable for all subjects, though tailored to the comfort of the individual subject. Each subject held a wooden drumstick, 21.5 cm in length, 1 cm in diameter, and weighing 10 g, which was to be swung rhythmically. Kinematic

Table 1
Number of Mod 1 and Non-Mod 1 Resonances From a Given W-Expected and Percentage of the K = 1 Line Covered by Them

		Number of ratios		Percentage at *K* = 1	
Level	*W*-expected	Mod 1	Non-Mod 1	Mod 1	Non-Mod 1
1	1:2	8	22	12.0	11.44
2	1:3	6	23	5.04	15.18
2	2:3	6	23	5.04	15.18
3	1:4	5	24	5.5	16.32
3	2:5	5	24	5.5	16.32
3	3:5	5	24	5.5	16.32
3	3:4	5	24	5.5	16.32
4	1:5	3	26	2.1	20.8
4	4:7	3	26	2.1	20.8
4	5:8	4	25	5.52	17.0
4	4:5	3	26	2.1	20.8

data were collected using a three-dimensional sonic digitizer (SAC, Westport, CT) and kinematic analysis software (Engineering Solutions, Columbus, OH). For collecting motion data on objects oscillated by hand, high-frequency sound emitters (30 mm long and 5 mm wide) were attached at the end of the objects. The sounds they emitted were detected by four microphones. In the present experiment the microphones were positioned at the corners of a square grid (77 cm by 77 cm) arranged vertically and placed at a distance of 40 cm in front of the tip of the drumstick when held in the subject's hand. The digitizer calculated the distances of the emitter from each microphone, using the three least noisy records to pinpoint the position of the emitter in three-dimensional space at the time of the emission. The signal was sampled at 90 Hz, passed through an analog/digital converter, and stored on an 80286-based PC hard disk for further software analyses.

Procedure. At the start of each trial, the subject was presented an initial metronome frequency to track with oscillatory motions of the drumstick in the right hand in a plane parallel to the subject's sagittal plane. After 15 s of performing the prescribed frequency, this initial, tracking frequency was terminated (the metronome was switched off), and the subject continued the midair drumstick motion at the initial frequency while a different frequency was presented through the earphones. With respect to the assigned task (namely, rhythmically moving the drumstick at the initially given frequency), this new audible frequency was irrelevant and could be ignored. The start of the trial proper began 10 s after the introduction of the new irrelevant frequency over the earphones. In light of the assumptions of phase entrainment, the phase at which the new frequency was introduced was not controlled. (Inspection of the time series of individual trials indicated that performance equilibrated to a particular ratio within the first 10 s under the combined rhythmic context; this was the case for Experiments 1, 2, and 3.) A trial lasted 70 s: The irrelevant frequency was played for 70 s, during which the subject attempted to continue the drumstick rhythm at the initial frequency. At the termination of each trial the subject could rest briefly for 30 s while the data were being stored on disk.

Design. A total of six frequency ratios were tested: 1:2, 1:3, 2:3, 2:5, 3:5, and 3:4. A frequency ratio was defined by the initial drumstick frequency divided by the subsequent metronome frequency, that is, $f_{drumstick}$ (initial) $\div f_{metronome}$. There were two initial drumstick frequencies of 0.67 Hz and 1 Hz. If the initial drumstick frequency was 1 Hz, the subsequent metronome frequency had to be set at 3 Hz to produce a ratio of 1:3; to produce a ratio of 3:5, the subsequent metronome frequency had to be set at 1.66 Hz. The six ratios defined the *W*-expected values. Each subject received each ratio twice at each initial drumstick frequency for a total of 4 trials. Ratios and initial frequencies were presented in random order for a total of 24 trials. A complete session lasted approximately 60 min.

The time series of each trial was inspected to confirm that the initial drumstick frequency at the outset of a trial was at the prescribed pace of either 0.67 Hz or 1 Hz. All trials satisfied this requirement. For each trial of each subject, a value for *W*-actual—the ratio in the mean that the subject actually achieved—was obtained as a quantity characterizing the overall behavior during a trial. This was calculated as the ratio of the mean drumstick frequency to the metronome frequency calculated over the duration of a trial. For example, given the initial drumstick frequency of $f_{drumstick}$ (initial) = 0.67 Hz, an average drumstick frequency over the trial of $f_{drumstick}$ (average) = 0.75 Hz, and an auditory metronomic frequency during the trial of $f_{metronome}$ = 1.12 Hz, then for that trial, *W*-expected = 3:5 and *W*-actual = 2:3.

Results and Discussion

To arrive at the actual numbers to be used in the mod 1 versus non-mod 1 contrast, we derived several measures of *W*-actual performance for each *W*-expected for each subject following the strictures laid down in the introduction to the experiment. The identity of these measures and their averages across subjects and metronome conditions are presented in Table 2. For example, of all trials prepared as a 2:3 *W*-expected, only 57% of all *W*-actuals were accepted as some resonance from the top six levels of the Farey tree. Synonymously, 43% of all *W*-actuals were outside the tolerance window of any ratio and were, therefore, unparseable. With respect to those parsed as some ratio, 17% of all trials remained in the prepared *W*-expected; 2% shifted to 1:1; 15% shifted to 1:2; and 23% of all trials shifted to some other ratio. With respect to the 23% of all trials for which a shift occurred from 2:3 to a ratio other than 1:1 or 1:2, 18% of all trials were mod 1 shifts, and 5% were non-mod 1 shifts. It is this latter comparison that constitutes the test of the unimodularity shift hypothesis.

Using the numbers computed in the manner of Table 2 for each subject as a function of metronome condition, we conducted a three-way analysis of variance (ANOVA) on independent variables of shift type (mod 1 vs. non-mod 1), initial drumstick frequency (0.67 Hz vs. 1 Hz), and *W*-expected (1:2, 1:3, 2:3, 2:5, 3:5, or 3:4). The dependent variable was the mean proportion of trials per subject exhibiting a shift of the specified type. There was a significant main effect of shift type (mod 1 = 18.5% vs. non-mod 1 = 5.5%), $F(1, 24) = 55.82$, $p < .0001$. Initial drumstick frequency did not significantly affect shift behavior, $F(1, 24) = 0.79$, $p > .05$, nor was there any main effect of *W*-expected, $F(5, 120) = 1.68$, $p > .05$. It may be concluded that more mod 1 shifts than non-mod 1 shifts occurred, contrary to an expectation based on the greater theoretical availability of non-mod 1 ratios and in agreement with the hypothesis that an intended manual rhythm f_{hand} will be deflected (spontaneously) by an auditory rhythm $f_{metronome}$ in the direction of forming a ratio $f_{hand}:f_{metronome}$ governed by the unimodularity structure of the Farey tree.

Table 2
Measures of Performance of W-Actual as a Function of W-Expected in Terms of Percentage of All Trials Averaged Across Subjects and Initial Drumstick Frequency in Experiment 1

	W-expected					
Measure	1:2	1:3	2:3	2:5	3:5	3:4
At any resonance	65.0	47.0	57.0	51.0	64.0	57.0
Remaining at *W*-expected	48.0	22.0	17.0	12.0	3.0	7.0
Shifts to 1:1	1.0	0.0	2.0	0.0	3.0	11.0
Shifts to 1:2	—	2.0	15.0	12.0	36.0	6.0
To another resonance	16.0	23.0	23.0	27.0	22.0	33.0
Mod 1 shifts	13.0	20.0	18.0	21.0	15.0	24.0
Non-Mod 1 shifts	3.0	3.0	5.0	6.0	7.0	9.0
Unparseable	35.0	53.0	43.0	49.0	36.0	43.0

Experiment 2

The focus of the second experiment was bimanual production of polyrhythmlike movements. Analogous to the task of Experiment 1, the present task did not require the subject to attend explicitly to the relation between the two rhythms composing the rhythmic pattern. The procedure was a version of Paradigm B and exploited the facility of human subjects to oscillate a pendulum at a stable frequency determined primarily by the length of the pendulum and referred to as the subject's *comfort-mode frequency* (Kugler & Turvey, 1987). While simultaneously producing an autonomous comfort-mode frequency with the pendulum hand, a subject was free to focus attention on the other hand (which was being used to track rhythmically a metronome). The task involved a minimal degree of attention toward the pendulum hand, just sufficient to maintain its continuous, rhythmical motion.

Previous investigations involving a 1:2 task (e.g., one hand tapping twice as fast as the other) have yielded results suggesting a central role for attention. Laterality effects with respect to handedness were amplified when attention was not focused on the dominant limb (Ibbotson & Morton, 1981; Peters, 1977, 1980, 1981, 1985a, 1985b, 1987). Laterality effects, however, were not obtained in a polyrhythmic (2:3) tapping task, provided that attention was focused on the faster hand, regardless of its relative dominance (Peters & Schwartz, 1989). In Experiment 2, the subject directed attention toward the faster hand (which tracked the metronome), while the dominant (right) hand remained free to execute the relatively autonomous task of comfort-mode oscillation. Depending on the frequency of the metronome with respect to the comfort-mode frequency of the pendulum, various *W*-expected ratios could be generated. The resultant performance due to bimanual coupling was investigated to determine whether subjects would maintain an initial bimanual ratio (*W*-expected) or would modify the frequency of the comfort-pendulum oscillation to settle at a different ratio (*W*-actual) in unimodular relation to *W*-expected.

Method

Subjects. A total of 16 right-handed subjects (10 men and 6 women) participated. The subjects were graduate and undergraduate students at the University of Connecticut; 7 were recruited from the university's music department (of whom 5 were percussion majors and 2 were guitar majors).

Materials. The same sonic digitizer and allied software used in Experiment 1 were used in the second experiment. A specially designed chair with leg rests permitted raising the legs toward the horizontal to avoid interference with the sonic data collection. Arm rests permitted comfortable support of both left and right wrists during oscillatory movements. The microphones of the three-dimensional sonic digitizer were placed horizontally at a vertical distance of 60 cm below the subject's chair. There were two pendulums (for the right hand) of 53 cm and 33 cm, both cut from wooden dowels 2.2 cm in diameter. Each included a molded plastic grip at one end. The two pendulums weighed 170 g and 131 g, respectively, with moments of inertia for rotation about an axis parallel to the ground plane and through a point in the wrist of 52,309 g cm^2 and 165,348 g cm^2, respectively. The dimensions of the wooden drumstick (for the left hand) and the position of the metronome behind the subject were identical to those used in Experiment 1.

Procedure and design. After settling into the data collection chair, the subject was shown how to rest each wrist at the end of the arm rests of the chair. The subject was then given the longer pendulum in the right hand and asked to oscillate the pendulum parallel to the saggital plane at his or her most comfortable rhythm, so as to become familiar with the concept of a comfort-mode frequency. The subject was then handed the small drumstick in the left hand and shown how to make midair oscillatory movements parallel to the plane of motion of the pendulum. When the subject had moved both hands independently, the experimenter asked the subject to begin oscillating the pendulum in the right hand at a comfort-mode frequency and to attend to the left hand and to begin tracking a metronome of a different frequency while maintaining the movement of the pendulum. The subject was presented several different right-hand/left-hand frequency ratios with both long and short right-hand pendulums for approximately 10 min until he or she had demonstrated a capacity for such bimanual motion. In addition, it was strongly emphasized that they should track the metronome with the left-hand drumstick as closely as possible, even if the right-hand pendulum changed its initial comfort-mode frequency. Any subject who could not maintain the bimanual motion while tracking the metronome was asked to leave the experiment.

Research has shown that the elected comfortable frequency at which a person swings a hand-held pendulum approximates closely the gravitational eigenfrequency of the pendulum as given by $f = (\frac{1}{2}\pi)(g/L)^{\frac{1}{2}}$, where L is the equivalent simple pendulum length (see Kugler & Turvey, 1987; Rosenblum & Turvey, 1988). Accordingly, given two hand-held pendulums of different lengths, as in the present experiment, the two comfort frequencies will be reliably different. For each subject, the comfort-mode frequency for each of the two pendulums was determined as the mean of three trials, each of 30 s duration. During each trial, the average frequency was calculated from the sum of all cycle frequencies; each cycle frequency was measured as the inverse of the duration from one peak extension to the next. The comfort-mode, pendulum frequency was then used to determine a frequency for the metronome-tracking hand whereby the left hand always took the higher frequency of a ratio in comparison with the comfort-mode frequency.

A total of eight ratios (*W*-expected values) were tested, namely, 1:2, 2:3, 2:5, 3:5, 3:4, 4:7, 5:8, and 4:5. For each trial, the subject began by swinging the pendulum comfortably for 15 s. At this point the metronome was switched on, and the subject began oscillating the drumstick. Ten seconds later, the trial proper and data collection began; subjects proceeded to track the metronome with the oscillations of the drumstick while simultaneously swinging the pendulum for a trial duration of 30 s. If the drumstick oscillations did not match the metronome beat, then the trial was discarded and repeated again. Three subjects (not included in the 16 tested) could not perform the bimanual task and were asked to leave the experiment. Subjects could rest at the end of a trial for 30 s while data were stored on disk. Each *W*-expected was tested twice with each of two pendulum lengths for a total of four trials. *W*-expecteds were randomized and presented in alternating blocks of eight trials, beginning with the long pendulum. All trials within a block involved the same length of pendulum. At the start of each block the comfort mode for the new pendulum was obtained as the average of three test trials. A complete session lasted approximately 60 min.

Results and Discussion

As in Experiment 1, a value for *W*-actual characterized the overall behavior during a trial and was calculated as the ratio of the average right-hand (comfort-mode pendulum) frequency f_{right} (average) to the left hand (metronome tracking) frequency f_{left}. Because of the conditions of the experiment, *W*-actual was always calculated for trials in which f_{left} did not deviate in the mean (and deviated minimally on a cycle-to-cycle basis) from the metronome frequency. Any deviations of *W*-actual from *W*-expected, therefore, were due to f_{right} (average) $\neq f_{right}$ (initial), meaning that the pendulum oscillations had shifted to a frequency other than the comfort-mode frequency.

Assignments to *W*-actual ratios were through the circle map conventions identified in the introduction of Experiment 1. Table 3 presents the proportions of mod 1 and non-mod 1 shifts (averaged over subjects, skill, and pendulum conditions) with their derivations for each *W*-expected. A four-way ANOVA was performed with between-subjects variable of skill (nonmusicians vs. musicians) and within-subjects variables of pendulum length (45 cm vs. 26 cm), shift type (mod 1 vs. non-mod 1) and *W*-expected (1:2, 2:3, 2:5, 3:5, 3:4, 4:7, 5:8, and 4:5). The dependent measure, as in Experiment 1, was the mean proportion of trials per subject exhibiting either a mod 1 or a non-mod 1 shift determined, in the manner of Table 3, for each of the experimental conditions. The main effect of shift type was significant (mod 1 = 13.52% vs. non-mod 1 = 7.61%), $F(1, 14) = 11.31$, $p < .01$, but the main effects of *W*-expected, pendulum, and skill were not, $F(7, 98) = 0.65$, $p > .05$, $F(1, 14) = 0.04$, $p > .05$, and $F(1, 14) = 0.02$, $p > .05$, respectively. There was also a significant interaction between shift type and ratio, $F(7, 98) = 2.78$, $p < .05$, due to an increase in non-mod 1 shifts over mod 1 shifts for two of the ratios tested, namely, 4 : 7 and 4 : 5. Despite the contrary patterning for these latter two ratios, the general conclusion to be drawn from Experiment 2 is consistent with that from Experiment 1: When a resonance shift occurred, it tended to be toward a neighboring ratio predictable from the unimodular structure of the Farey tree.

Experiment 3

When two rhythmic movement patterns are drawn from the same level of the Farey tree, are they necessarily alike in their dynamics? Inspection of Figure 2 in conjunction with Figure 3 reveals that for any given coupling strength $K < 1$, the widths of the Arnold tongues for same-level ratios are identical and, in fact, symmetrical. The empirical relation between width of Arnold tongue and oscillation stability implies that rhythmic movement patterns from a common Farey level will have similar dynamics in terms of fluctuations and tendency to shift to another ratio. However, other considerations point to differences among same-level ratios and lead, therefore, to expectations of asymmetrical dynamics.

A *Fibonacci ratio* is one in which addition of the ratio's numerator and denominator yields the denominator of the following ratio in a series of ratios known as the Fibonacci series; the ratio's denominator yields the numerator of the next ratio in the Fibonacci series. Beginning with 1:1 at Level 0, the Fibonacci series to Level 4 is 1:2, 2:3, 3:5, and 5:8. The Fibonacci series provides increasing rational approximations to the irrational number known as the "golden mean" $[(\sqrt{5} - 1)/2 = 0.618\ldots]$, where an irrational number has nonrepeating digits following the decimal point (e.g., π). The most explicit representation of both rational and irrational numbers is through the method of "continued fractions" (Hardy & Wright, 1938; Schroeder, 1984, 1991). As can be seen in Figure 4, the simple ratios have the shortest continued fraction representation, whereas the Fibonacci ratios have the longest continued fractions with entries consisting of only ones. This implies that a continued fraction for a Fibonacci ratio takes longer to converge than for a non-Fibonacci from the same Farey level. The continued fraction representation of the golden mean irrational consists of a nonterminating series of ones and justifies its label as the most irrational of irrational numbers (Schroeder, 1991). Experiments directed at quasiperiodicity have exploited this property of Fibonacci ratios to avoid mode locking. By tuning the winding number, *W*, of a dynamical system to be close to a higher order Fibonacci ratio, quasiperiodic rather than mode-locked dynamics have been observed (Fein et al., 1985; Stavans, 1987). In other words, it is easier to avoid mode locking if the system

Table 3
Measures of Performance of W-Actual as a Function of W-Expected in Terms of Percentage of All Trials Averaged Across Subjects and Pendulum Length in Experiment 2

Measure	*W*-expected 1:2	2:3	2:5	3:5	3:4	4:7	5:8	4:5
At any resonance	72.22	64.48	44.84	63.88	45.82	55.75	63.68	43.64
Remaining at *W*-expected	53.57	14.09	8.73	4.56	3.17	4.96	5.95	0.0
Shifts to 1:1	0.0	1.78	0.0	0.0	8.13	1.39	1.78	15.87
Shifts to 1:2	—	28.17	11.9	38.09	18.25	30.75	24.60	9.52
To another resonance	18.65	20.44	24.21	21.23	16.27	18.65	31.35	18.25
Mod 1 shifts	10.91	14.48	21.43	18.05	13.09	5.95	20.04	4.17
Non-mod 1 shifts	7.74	5.96	2.78	3.18	3.18	12.70	11.31	14.08
Unparseable	27.78	35.52	55.16	36.12	54.18	44.25	36.32	56.36

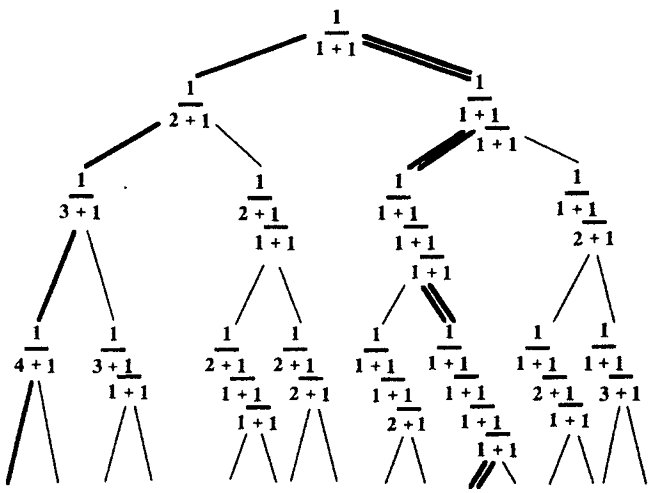

Figure 4. Continued fraction representation of ratios from Levels 1 to 4 of the Farey tree. (Note the path of ratios with fastest versus slowest convergence properties: simple ratios [left-hand heavy line] vs. Fibonacci ratios [double heavy line].)

is tuned close to an irrational number, and it is easiest if this number is a Fibonacci with maximal convergence time. In Figure 2, quasiperiodic motion is obtained by covarying W-expected and K to determine a point in the black, quasiperiodic region that avoids mode locking. However, as mentioned in the introduction, as K increases, the size of the mode-locked regions correspondingly grows such that, experimentally, it actually becomes very difficult to avoid mode locking. Therefore, to observe a shift from a non-mode-locked (quasiperiodic) ratio to a mode-locked ratio, the best choice of W-expected would be a Fibonacci ratio. Furthermore, because the Farey geometry is assumed to constrain shift dynamics, it is expected that more mod 1 shifts will occur than non-mod 1 shifts. And because the numerical substructure of the Fibonacci ratios is maximally complex, it is expected that more mod 1 shifts will occur for Fibonacci than for non-Fibonacci ratios. We refer to this as the *continued fraction substructure hypothesis.*

In the third experiment, we compare the "most simple" and the "most complex" ratios at a given Farey level, as identified through the number-theoretic construction of Figure 4. More specifically, we use a version of Paradigm B to compare the single simple rhythmic pattern from a given Farey level with the single Fibonacci polyrhythmic pattern from that same level. If the complexity of a ratio's substructure is a determinant of its dynamics, these same-level W-expecteds should not be associated with the same pattern of persistence and change, that is, the pattern of mod 1 and non-mod 1 shifts should be different for ratios from the same level. It is to be expected that given the long convergence rates of the associated continued fraction substructure, a Fibonacci ratio will engender more mod 1 shifts than a simple ratio when both are drawn from the same level in the Farey tree. This number-theoretic constraint may provide a basis for the observed asymmetry in performance dynamics of the underlying symmetrical Arnold tongues.

Method

Subjects. A total of 13 right-handed undergraduate subjects (8 men and 5 women) from the University of Connecticut participated. Five subjects had at least 5 years experience in playing a musical instrument, either through coursework or recreationally.

Materials. The same data collection chair as in Experiment 2 was used. There were two left-hand pendulums and one right-hand pendulum. Each pendulum consisted of an aluminum rod; a mass was attached at the lower end of the left pendulums but not the right pendulum. Both left-hand pendulums were 58 cm long and 1.3 cm in diameter, with shafts weighing 249 g. Each had either a heavy (500 g) or light (100 g) mass attached at its end. The corresponding moments of inertia of the two left-hand pendulums (for rotation about a point in the wrist in a plane perpendicular to the ground plane) were 621,274 g cm^2 and 1,813,203 g cm^2, respectively. The right-hand pendulum was 36 cm long and 1.3 cm in diameter, with mass of 167 g and moment of inertia of 64,128 g cm^2.

Procedure and design. The subject was required to coordinate a pendulum at a comfort-mode frequency in the left hand while tracking a metronome with another pendulum in the right hand. In Experiment 2, the right-hand wrist-pendulum provided the comfort-mode frequency, while the left hand tracked the metronome at a higher frequency than the right. In Experiment 3, because of the nature of the simple ratios used, the tracking hand was required to oscillate at considerably higher frequencies than in Experiment 2. Pilot studies indicated that a subject could only maintain such high frequencies if the dominant right hand tracked the faster metronome frequency while the left hand oscillated at the lower, comfort-mode frequency.

To familiarize the subject with the task, he or she was given the long pendulum in the left hand and asked to oscillate it freely in a plane parallel to the body's saggital plane (in order to become familiar with his or her comfort-mode frequency). The subject was then handed the shorter pendulum in the right hand and asked to track a metronome while simultaneously maintaining rhythmic movement of the left pendulum. Each subject demonstrated competence with several different right-hand/left-hand frequency ratios with both the heavy and light masses attached to the left (comfort mode) pendulum. It was emphasized that the subject should track the metronome with the right-hand pendulum as closely as possible, even if the left-hand pendulum altered its initial frequency. Any subject who could not demonstrate sufficient competence of the bimanual task was eliminated from further experimentation. This occurred with only 2 subjects.

A total of six W-expected ratios (three simple and three Fibonacci), drawn from three Farey levels, were tested: 1:3 versus 2:3, 1:4 versus 3:5, and 1:5 versus 5:8. During a trial, the subject swung the left pendulum at a comfort-mode frequency for 15 s. The metronome was then switched on, the subject began oscillating the right-hand pendulum, and after a further 10 s, data collection and the trial proper began. During the 30-s trial, the subject simultaneously tracked the metronome with right-hand pendulum oscillations while oscillating the left-hand pendulum. If the right pendulum oscillations did not match the metronome beat, the trial was terminated and repeated. At the end of a trial the subject rested for 30 s while data were stored on disk. The ratios were randomized and presented in two blocks of 6 trials for a total of 12 trials. At the start of each block the comfort mode for the left-hand pendulum was obtained as the average of two trials, each lasting 30 s. The first 6 subjects tested received the 500-g left-pendulum condition; the remaining 7 subjects received the 100-g left-pendulum condition. A complete session lasted approximately 30 min.

Results and Discussion

W-actual for each trial was calculated as the ratio of the average left-hand comfort-mode pendulum frequency to the average right-hand metronome-tracking pendulum frequency. The proportion of mod 1 and non-mod 1 shifts, with the other performance measures for each *W*-expected (all averaged over subjects and pendulums), are presented in Table 4. To test both the unimodularity shift hypothesis and the continued fraction substructure hypothesis, we conducted a five-way ANOVA with between-groups variables of pendulum and skill, and within-groups variables of shift type (mod 1 vs. non-mod 1) and ratio type (simple vs. Fibonacci). To determine whether there was an influence from Farey level of *W*-expected, we included an independent variable of Farey level of *W*-expected (Levels 2, 3, and 4), given that two *W*-expecteds were drawn from each of the three Farey levels. The dependent measure was the mean proportion of trials that made a shift of the specified type. There was a significant main effect of shift type (mod 1 = 12.15% vs. non-mod 1 = 2.67%), $F(1, 9) = 7.59, p < .05$, and of ratio type (simple = 2.5% vs. Fibonacci = 12.33%), $F(1, 9) = 7.12, p < .05$. Thus, if shifts occurred, they tended to be from Fibonacci ratios toward neighboring, unimodular ratios rather than from simple ratios toward nonneighboring ratios. Neither pendulum nor Farey level were significant, $F(1, 9) = 0.15$, $p > .05$, and $F(2, 18) = 0.28, p > .05$, respectively. Thus, an equivalent proportion of mod 1 and non-mod 1 shifts occurred for *W*-expecteds from all three Farey levels tested.

The main effect of skill was also significant (nonmusicians = 12.22% vs. musicians = 2.60%), $F(1, 9) = 6.13, p < .05$. There was also a significant interaction between skill and shift type (mod 1: nonmusicians = 22.22% vs. musicians = 2.08%; non-mod 1: nonmusicians = 2.22% vs. musicians = 3.12%), $F(1, 9) = 9.34, p < .05$. The simple effects analysis indicated that the two skill groups differed in the proportion of mod 1 shifts that occurred; there were more mod 1 shifts for the nonmusicians than for the musicians ($p < .01$). Regarding the non-mod 1 shifts, equal proportions occurred for both skill groups. In addition, the adjacency structure of resonances did not seem to determine a musician's preference for mod 1 shifts over others, and a shift to an adjacent ratio was as probable as a shift to a more distant one. In contrast to the musicians, more mod 1 than non-mod 1 shifts tended to occur for the nonmusicians ($p = .052$).

The skill distinction may be tied to the effects of inertia exerted through the dynamical dispositions of a rhythmic movement unit. A rhythmically moving limb or limb segment can be viewed as a hybrid oscillator composed of a harmonic dynamic (associated with the pendular dimensions of the limb) and a relaxation dynamic (associated with neuromuscular and metabolic control processes; Beek & Beek, 1988; Rosenblum & Turvey, 1988). A harmonic dynamic is stable in time; a relaxation dynamic is less so. When coupled, the component dynamics interact such that the relaxation dynamic may be "tuned" by the harmonic dynamic. In turn, the harmonic dynamic may be "detuned" by the relaxation dynamic. Smaller pendular dimensions raise the possibility of more pronounced contributions from the relaxation dynamic and a consequent reduction in temporal stability. In the present experiment, the rotational inertias of the two pendulums were sufficiently large to guarantee a marked influence of the harmonic dynamic in both cases; no differential effects due to the pendulum were observed. The musicians did not shift according to the unimodular structure of the Farey tree. Thus, the pendulum's relatively strong harmonic component may have provided additional tuning of the musicians' relaxation dynamic above any tuning related to intrinsic rhythmic competence. Unlike the musicians, the nonmusicians made more mod 1 shifts than non-mod 1 shifts from *W*-expected. This could have been due to the nonmusicians' lack of specific rhythmic competence as embodied in the relaxation component; precisely because of such absence, their shifts were free to be attracted by adjacent, unimodular resonances.

Separate four-way ANOVAs were conducted for the two skill groups with independent variables as before. For the nonmusicians, shift type was significant (mod 1 = 22.22% vs. non-mod 1 = 2.22%), $F(1, 6) = 28.73, p < .01$, as was ratio type (simple = 5% vs. Fibonacci = 19.44%), $F(1, 6) = 14.98, p < .01$. There was also a significant interaction between shift type and ratio type (mod 1: simple = 8.89% vs. Fibonacci = 35.55%; non-mod 1: simple = 1.11% vs. Fibonacci = 3.33%), $F(1, 6) = 7.09, p < .05$. The simple effects analysis indicated that more mod 1 than non-mod 1 shifts occurred with the Fibonacci ratios ($p < .01$) and that

Table 4
Measures of Performance of W-Actual as a Function of W-Expected in Terms of Percentage of All Trials Averaged Across Subjects and Pendulum Mass in Experiment 3

Measure	*W*-expected					
	1:3	2:3	1:4	3:5	1:5	5:8
At any resonance	80.41	74.37	56.25	36.25	22.08	36.04
Remaining at *W*-expected	72.08	55.83	51.25	4.79	20.42	4.79
Shifts to 1:1	0.0	0.0	0.0	0.0	0.0	0.0
Shifts to 1:2	0.0	0.0	0.0	0.0	0.0	7.29
To another resonance	8.33	18.54	5.0	31.46	1.67	23.96
Mod 1 shifts	6.67	12.29	5.0	25.0	1.67	22.29
Non-mod 1 shifts	1.67	6.25	0.0	6.46	0.0	1.67
Unparseable	19.59	25.63	43.75	63.75	77.92	63.96

the ratio types were distinct in terms of mod 1 shifts only ($p < .05$). That is, an equivalent number of non-mod 1 shifts occurred for both simple and Fibonacci ratios. No significant effects were revealed for the four-way ANOVA conducted on the musician data.

By way of summary, the focus is returned to the specific question to which Experiment 3 was directed: When two rhythmic movement patterns are drawn from the same level of the Farey tree, are they necessarily alike in their dynamics? On the basis of the preceding analyses, the answer is no. However, we must be circumspect about the generality of this answer, because the experiment contrasted only maximally simple and maximally complex (Fibonacci) ratios. The experiment did not provide any evidence about differences among complex rhythms but only about a level's most complex rhythm in relation to its most simple rhythm. Nor did Experiment 3 provide evidence on the potential inequality between a level's simple ratio and its non-Fibonacci complex ratios. Future experiments can be directed at these other aspects of the question posed above. In the meantime, we can turn to Experiments 1 and 2 for some measure of the likelihood that continued fraction differences among ratios translate into dynamical differences among rhythmic movement patterns bearing those ratios. Experiments 1 and 2 included Fibonacci ratios and both simple and complex non-Fibonacci ratios from the same Farey level. The data from those experiments were examined using one-way ANOVAs. The results, summarized in Table 5, indicate that movement patterns with Fibonacci ratios do indeed differ on measures of persistence and change from movement patterns with non-Fibonacci ratios, both simple and complex. Thus, the results of Table 5 lend support to the continued fraction hypothesis that ratios from the same level may exhibit different dynamics on the basis of their numerical substructure.

General Discussion

In this article, we have used procedures that allowed us to impose simple rhythmic patterns (e.g., 1:2 and 1:3) or complex, polyrhythm-like patterns (e.g., 2:3 and 3:5) on the movement systems of human subjects. The significant feature of these procedures is that the subject executed the experimentally desired patterns implicitly rather than explicitly. This was achieved by defining the subject's focal task as a simple oscillatory motion of one limb segment and introducing as a backdrop for this focal task another periodic process that was, from the perspective of executing the focal task, subsidiary and perhaps even irrelevant. In Experiment 1 the focal task was the periodic motion of a hand-held object to a memorized rhythm, and the subsidiary task was listening to the beat of a metronome. In Experiments 2 and 3 the focal task was the periodic motion of a hand-held object at a pace dictated by a metronome, and the subsidiary task was maintaining a pendulum oscillation at the subject's comfort-mode frequency. Using the frequency of either the focal periodic process (Experiment 1) or the subsidiary periodic process (Experiments 2 and 3) as the baseline frequency, we could then choose the frequency of the other periodic process such that the two periodic processes would form either a simple or a complex rhythm. It could be said that through these procedures, we "prepared" the human movement system at the outset of a trial in a given rhythmic pattern and then observed the ability of that person's movement system to sustain the given pattern. Although these rhythmic movement patterns, referred to as *W*-expected, were only implicitly defined on the movement system (the subject neither intended them nor knew the ratios that they composed), it is nonetheless the case that their maintenance depended on the dynamics of interacting (neural and neuromuscular) subsystems. Our research was based on the assumption that these simple experimental tasks tap the natural dispositions of the human movement system to settle into a relatively optimal coordination. Failures to preserve initially prepared patterns would, therefore, be revealing of dynamical principles underlying the assembling, sustaining, and selection of rhythmic coordination patterns.

Table 5
Significant Differences in Percentage of Mod 1 Shifts Between Fibonacci and Non-Fibonacci Ratios Across Experiments 1–3 as a Function of Pendulum Characteristics ($p < .05$)

Measure	Fibonacci		Non-Fibonacci
Experiment 1			
1 Hz	3:5	<	3:4
Experiment 2			
Long	5:8	>	4:7
Long	5:8	>	4:5*
Short	5:8	>	4:5
Experiment 3			
500 g	5:8	>	1:5
100 g	3:5	>	1:4

* $p = .054$.

The unimodularity hypothesis states that simple and complex rhythmic movement patterns are constrained by a layout of resonances ordered by the Farey tree. On those trials in which a subject failed to preserve the rhythm in which he or she was initially prepared, the experimental evidence reveals that the subject was more likely to exhibit a rhythm that related to the prepared ratio in a unimodular rather than non-unimodular fashion. As outlined in the Appendix, the geometrical interpretation of a unimodular shift is that it is optimal in the sense of preserving invariant the translatory symmetry underlying the transformation. In summary, performance conformed to the resonance layout: Whenever an initially prepared rhythm failed to be sustained, the shift tended to be toward neighboring, unimodular ratios.

Continuous with numerical studies of circle map dynamics (Artuso, Cvitanovíc, & Kenny, 1989; Cvitanovíc, Shraiman, & Söderberg, 1985), the continued fraction hypothesis states that the performance of a rhythmic movement pattern is affected not only by the geometry of unimodular ratios but also by the complexity of the prepared ratio as indexed by its continued fraction substructure. The hypothesis suggests, for example, that simple rhythms distinguish from complex rhythms because their simpler continued fraction substructure implicates a simpler nesting of rhythmic subtasks for their execution (see Schmidt et al., 1991). It also suggests that

Fibonacci complex rhythms will distinguish from non-Fibonacci complex rhythms for the same reason. Experimentally, it was shown that rhythms from the same Farey level were not sustained with the same probability. In Experiment 3, a subject initially prepared in a simple ratio from a given Farey level tended to make fewer shifts to mod 1 ratios than when prepared in a Fibonacci complex ratio from the same level. Examination of the prepared complex rhythms of all three experiments revealed a nonuniformity in the prevalence of shift behavior among rhythms from the same Farey level. In summary, the results of Experiments 2 and 3 indicate that Fibonacci rhythms were preserved less well than non-Fibonacci rhythms, consistent with the fact that Fibonacci ratios have a maximally complex continued fraction substructure.

Reexamining Previous Research

Previous explanations of both unimanual and bimanual rhythmic performance have depended on arbitration by internal codes as the prime coordinating candidate. The assumption that unimanual performance of a nonisochronous rhythm involves independent timekeepers was tested using the task of tapping a serial rhythm composed of two successive durations related either as simple (1:2, 1:3, or 1:4) or as complex (2:3, 2:5, 3:4, 3:5, or 4:5) ratios (Povel, 1981). It was concluded that the durations were not independently coded by the subject but integrated in some manner. During reproduction in a continuation paradigm, a subject spontaneously modified interpulse ratios below .5 toward .5 and ratios greater than .5 toward 1.0. The following results from Povel (1981, Table 2) indicate that shifts from expected to actual ratio may be compatible with unimodular bifurcation sequences in the Farey tree (the second ratio shifted to refers to a pattern of longer cycle time): .25 to .33 or .33 (1:4 to 1:3); .33 to .37 or .39 (1:3 to 2:5); .4 to .45 or .44 (2:5 remained 2:5); .5 to .48 or .47 (1:2 remained 1:2); .6 to .48 or .51 (3:5 to 1:2); .66 to .49 or .55 (2:3 to 1:2); .75 to .63 or .66 (3:4 to 5:8 or 2:3); and .8 to .74 or .72 (4:5 to 3:4). Similarly, in the tapping data of Summers, Hawkins, and Mayers (1986, Table 2), the ratio shifts for either musicians or nonmusicians were .25 to .31 or .36 (1:4 to 1:3); .33 to .39 or .41 (1:3 to 2:5); .5 to .48 or .46 (1:2 remained 1:2); .66 to .52 or .54 (2:3 to 1:2); and .75 to .61 or .71 (3:4 to 3:5 or 3:4). These results were interpreted as evidence that intervals are perceived and produced according to Gestalt processes of "distinction" and "assimilation," which either lengthen (toward 1:2) or shorten (toward 1:1) the intervals between events (Fraisse, 1978, 1982, 1987; Povel, 1981; Summers, Hawkins, & Mayers, 1986; Summers, Bell, & Burns, 1989). These results are also consistent with the unimodular geometry of resonances, which may provide a simpler and more general account.

The data from Deutsch (1983) on bimanual polyrhythm tapping involved a dependent measure (fluctuation in performed ratio) different from the present analysis but may also support the resonance constraints hypothesis. As *W*-actual is drawn from higher order levels in the Farey tree (as in Deutsch's experiment), width and stability of the resonance decreases, and correspondingly, fluctuations in performance will increase. Additionally, Deutsch's data reveal a systematic distinction in the performance stability of Fibonacci (3:5) and non-Fibonacci (2:5) polyrhythms (see also Peper et al., 1991).

Relevant to the unimodularity hypothesis and the tasks used here is the finding that a rhythm not explicitly controlled by the subject, the respiratory cycle, may become mode locked to the locomotory cycle. In seasoned marathon runners, these cycles are mode locked not only at 1:2 (primarily) but also at 1:4, 1:3, 2:3, and 2:5, whereas novice runners exhibit no such entrainment (Bramble & Carrier, 1983). In addition, as speed increases, shifts occur between these ratios. In contrast, such cycles are limited to 1:1 in quadrupeds, possibly because of forces on the thoracic cavity (Alexander, 1989). Entrainment of respiration to locomotion has also been found in bicycle ergometry tasks (Kohl, Koller, & Jäger, 1981), in which a similar patterning of shifts was observed (Paterson et al., 1986). These shifts may permit a greater economy of effort for the pulmonary and locomotor systems (Paterson et al., 1986), because the metabolic cost of entrained cycling is significantly lower than that of nonentrained cycling (Garlando, Kohl, Koller, & Pietsch, 1985). We suspect that unimodular shifts, because they preserve invariant the geometry of mode locking (see the Appendix), may provide an efficient "gearing" of the rhythmical movement apparatus.

Dynamical Basis for Shifts in Complex Rhythms

Two possibilities exist to account for the method by which the observed shifts in coordination occurred. We have assumed that subjects perform in resonances corresponding to the region at or below the $K = 1$ line where the resonances do not overlap (Figure 2). For a shift to occur from a ratio at a higher order level to a more stable mod 1 ratio, it is necessary to "skip across" a less stable, narrower Arnold tongue from a higher order level of the Farey tree (see Figure 2). For example, when *W*-expected is drawn from Level 2 (e.g., 1:3 or 2:3), the Level 3 ratios, which have relatively strong attractors (wide Arnold tongues), may capture coordinations en route to a lower order ratio such as 1:2 or 1:1.

Also exploring Farey tree dynamics, Peper et al. (1991) made the alternative assumption that polyrhythm performance may involve a high value of coupling in the region where resonances overlap (i.e., $K > 1$). Their method of inducing shifts required a subject to tap continuously a bimanual polyrhythm as a pacing frequency was increased until a spontaneous bifurcation, analogous to a second-order phase transition (Kelso et al., 1987), occurred at another Farey ratio. Because of the overlapping resonances where $K > 1$, mode locking is unavoidable. However, theory indicates that this leads to deterministic chaos, that is, arbitrarily small changes in initial phase conditions will project the dynamical system into a different ratio depending on which ratios overlap. In contrast to such predictions, human coordination is relatively robust under conditions of small perturbations to phase, and chaotic dynamics are generally not observed. Experiments have shown that 1:1 mode locking may be maintained when the natural frequency ratio (Ω)

of the hand-held pendulums decreases. This occurs under the assumption that subjects perform in the widest 1:1 resonance, which, when K is increased to a value of approximately 6.5, traverses all Ω values above the $K = 1$ line (see Figure 2; Schmidt et al., 1991). Thus, although a Farey tree geometry appears to govern shift behavior, it is uncertain how this abstract geometry translates into actual transition mechanisms.

Concluding Remark

The present research suggests that resonance, as a general principle governing self-organized processes, may be applicable to the coordination of human movement within a polyrhythmic context (Beek, 1989; deGuzman & Kelso, 1991; Turvey, 1990). This suggestion is continuous with Gibson's (1966, 1979) suggestion that resonance may provide a better metaphor for perception–action than computation. An advantage of a physicalist explanation through resonance is that it cuts across the organism–environment boundary and advances an appreciation of the lawful mutuality permitting the coordination of organisms and their surroundings.

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Appendix

Geometrical Interpretation of Unimodularity

A lattice is a figure of lines and defines a figure of points. An infinite two-dimensional space of lines has points defined where the lines intersect and is called a *point lattice.* The lattice formed by lines drawn parallel to the rectangular coordinate axes such that it is divided into unit squares (each with an area = 1) is called the *fundamental lattice,* Λ. Any lattice point may be considered an origin, *O*, and the properties of the lattice are symmetrical about *O*. When two lattices determine the same point lattice, they are said to be equivalent. Figure A1 depicts the point lattice defined by the two equivalent lattices based on *OP*, *OR*, and *OR*, *OQ*, where *O*(0, 0), *P*(0, 1), *Q*(1, 1) and *R*(1, 2) are points on the Λ lattice.

We may associate numbers with each point of a point lattice such that each has coordinates (x, y). This set of numbers constitutes a modulus of numbers because the sum and difference of any two members of the set are themselves members of the set (Hardy & Wright, 1938). If *P* and *Q* are the points (x_1, y_1) and (x_2, y_2) of Λ, then any point of a lattice based on *P* and *Q* has the following coordinates:

$$x = mx_1 + nx_2, \qquad y = my_1 + ny_2,$$

where *m* and *n* are integers.

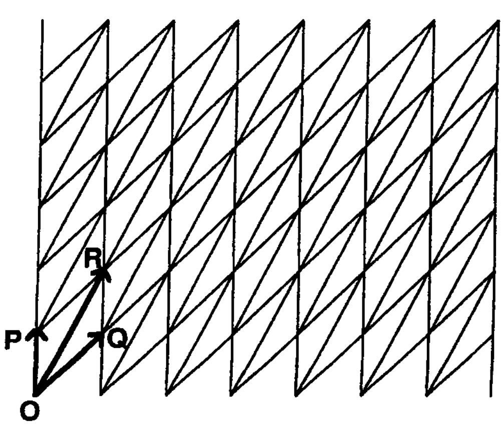

Figure A1. The point-lattice defined by the two equivalent lattices based on *OP*, *OR*, and *OR*, *OQ*, where *O*(0, 0), *P*(0, 1), *Q*(1, 1) and *R*(1, 2) are points on the fundamental lattice, Λ.

A transformation on the Λ lattice—

$$x' = ax + by, \qquad y' = cx + dy,$$

where *a*, *b*, *c*, and *d* are integers—transforms any point of the Λ lattice into some other point of the Λ lattice:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ab \\ cd \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}.$$

Solving for x and y,

$$x = \frac{dx' - by'}{ad - bc}, \qquad y = \frac{cx' - ay'}{ad - bc}.$$

If the denominator $ad - bc = \pm 1$, then integer values of x' and y' give integer values of x and y, and every point (x', y') of the Λ lattice corresponds to some other point (x, y) in Λ, that is, a one-to-one mapping. Under conditions where the denominator is ± 1, such transformations of the Λ lattice are called *unimodular* (Hardy & Wright, 1938). Thus, a matrix of integer elements with determinant $(ad - bc) = \pm 1$ is called a *unimodular matrix* (Weyl, 1980). If two equivalent lattices have bases *b*1 and *b*2, each consisting of linearly independent vectors, then such lattices are also connected by a unimodular transformation (Figure A1). Because the determinant of a transform indexes the area of the object, a unimodular transformation preserves area, and in this sense, the underlying geometry remains invariant: All lattice points are mapped onto other lattice points, and none are skipped.

If the Farey ratios *p/q* are defined as points on the fundamental lattice Λ, each with coordinates (p, q), then all such points are coprime and said to be *visible* from the origin because there are no intervening lattice points on a line drawn from *O* to (p, q). The set of all such visible points defines a convex set, in particular, a Minkowski "Strahlkorper" or "ray body" (Coxeter, 1961; Hardy & Wright, 1938). Two Farey ratios, *a:c* and *b:d*, corresponding to Points *P* (*a*, *c*) and *Q* (*b*, *d*) of Λ, define a new lattice based on *OP* and *OQ* that we call a "Farey lattice." If *P* and *Q* have a unimodular relationship, then the edges of the parallelograms based on *OP* and *OQ* demarcate convex sets such that every point of a set is visible from every other point in the set because there are no intervening Farey points. Conversely, a Farey lattice generated from non-unimodular basis vectors defines nonconvex sets consisting of non-visible points, because there are intervening points. Figure A2 depicts the fundamental lattice with superposed Farey points, together with the Farey lattice and parallelogram generated by ratios with either a unimodular (A) or a non-unimodular (B) relationship. As

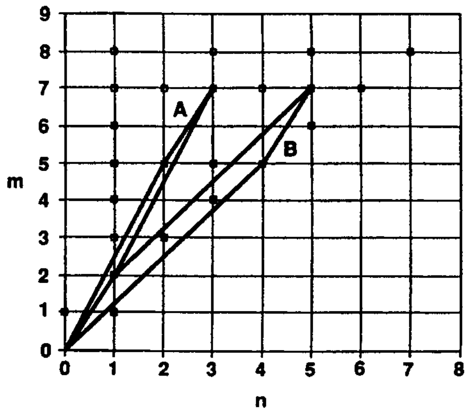

Figure A2. The fundamental lattice with superimposed Farey points is shown, together with the Farey lattices that would be generated by ratios having either unimodular relation—(1:2) and (2:5), as in A—or nonunimodular relation—(1:2) and (4:5) as in B.

shown, the number of intervening points equals the degree of modularity less one; that is, 1:2 and 2:3 are unimodular (2*2 − 1*3 = 1: no intervening Farey points) and 1:2 and 4:5 are non-unimodular (2*4 − 1*5 = 3: two intervening Farey points). The existence of intervening points implies that the lattice determined by two non-unimodular ratios is nonequivalent to the fundamental lattice Λ, and because the determinant of such a transform will be greater than unity, such a transformation can no longer be said to preserve as invariant the translatory symmetry relating both lattices (i.e., the non-unimodular transformations break symmetry). For the unimodular shifts, the Farey lattice has optimal form because such transformations determine the translatory symmetry operations that relate all points; such transformations constitute a basis or generator of the lattice symmetry and preserve closure of the lattice.

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